JOHN CURTIN College of the Arts
COLLEGE OF THE ARTS

YEAR 11 MATHEMATICS SPECIALIST

TEST 1, 2018

(Reasoning and Counting Techniques)

CALCULATOR-ASSUMED

Student's Name: Solutions

Total Marks: ~~44~~

Time Allowed: 45 mins

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

Standard Items: Pens, pencils, eraser, ruler

Special Items: Up to three approved calculators
One page (unfolded A4 sheet) front and back of Notes
WACE Formula Sheet

INSTRUCTIONS TO STUDENTS

Do not open this paper until instructed to do so.
You are required to answer ALL questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

It is recommended that students **do not use pencil**, except in diagrams.

1. [7 marks]

For the statement *"If a quadrilateral is a rhombus then its diagonals are perpendicular to each other"*, decide if it is true or false. True / False

Then write the following statements and decide if they are true or false:

Converse: If a quadrilateral's diagnols are perpendicular to each other then it is a rhombus. True / False

Inverse: If a quadrilateral is not a rhombus, then it's diagnols are not perpendicular to each other True / False

Contrapositive: True / False

If a quadrilateral's diagnols are not perpendicular to each other then it's not a rhombus

2. [5 marks]

Use proof by contradiction to prove that there are no positive integer solutions to the equation $x^2 - y^2 = 1$.

Assume $\exists x, y \in \mathbb{Z}^+$ s.t. $x^2 - y^2 = 1$

$$(x+y)(x-y) = 1$$

$$\begin{aligned} x+y &= 1 \\ \text{and } x-y &= 1 \end{aligned} \quad \text{(or)} \quad \begin{aligned} x+y &= -1 \\ \text{and } x-y &= -1 \end{aligned}$$

$\Rightarrow x = 1$, $y = 0$ (contradicts) or $x = -1$, $y = 0$ (contradicts.

Both cases contradict the assumption therefor. there are no positive integer solutions to $x^2 - y^2 = 1$.

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3. [1, 2, 2, 2 = 7 marks]

A team of four students is selected from a candidate group to fill the positions on an upper school student council committee. The candidate group consists of six Year 12 boys, four Year 12 girls, five Year 11 boys and five Year 11 girls. How many teams are possible if:

	Boys	Girls	
Yr 11	5	5	10
Yr 12	6	4	10
	11	9	20

(a) there are no restrictions

$^{20}C_4 = 4845$ ✓

(b) they must consist of two boys and two girls

$^{11}C_2 \times {}^{9}C_2 = 1980$ ✓✓

(c) they must have at least two Year 11 students

No Yr 11: $^{10}C_4 = 210$

One Yr 11: $^{10}C_3 \times {}^{10}C_1 = 1200$

At least two: $4845 - 210 - 1200 = 3435$ ✓✓

(d) they must contain exactly one person from each of the four categories of the candidates

$^{6}C_1 \times {}^{4}C_1 \times {}^{5}C_1 \times {}^{5}C_1 = 600$ ✓✓

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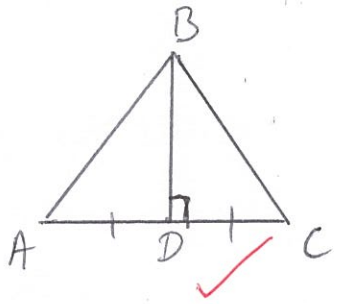
4. [4 marks]

Prove the following statement:

If in triangle ABC the perpendicular bisector of AC passes through B then ABC is an isosceles triangle.



in $\triangle ABD$ & $\triangle CBD$

$AD = AC$ (given)

$\angle BDA = \angle BDC$ (both 90°)

$BD = BD$ (common)

$\therefore \triangle ABD \cong \triangle CBD$ (SAS)

$\therefore AB = CB$ (matching sides equal)

$\therefore \triangle ABC$ is isosceles

5. [2, 2 = 4 marks]

a) Determine the least number of people needed to be in a group to ensure that at least 10 people have their birthdays in the same month.

$\underbrace{9 \times 12}_{\text{fill up the pigeon holes}} + 1 = 109$ people

b) A shop has a stock of 188 bottles of wine from six different producers. Prove that the stock includes at least 32 bottles from one of the producers.

$31 \times 6 = 186$

$\therefore$ The 187th and 188th bottles must go into a group to make 32.

By pigeon hole principle there must be at least 32 from one producer.

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6. [1, 2, 2, 2 = 7 marks]

A group of 7 people A, B, C, D, E, F and G have to stand in a line for a photograph. How many ways can they be arranged if

(a) There are no restrictions.

$7! = 5040$ ✓

(b) C must be in the middle.

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{C}\ \underline{3}\ \underline{2}\ \underline{1}$ ✓

$6! = 720$ ✓

(c) E and F are not next to each other.

E & F next to each other $= 6! \times 2! = 1440$ ✓

Not next to each other $= 5040 - 1440 = 3600$ ✓

(d) A is at the left end **or** G is at the right end.

$\underline{A}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} + \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{G} - \underline{A}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{G}$

$6! + 6! - 5!$ ✓

$= \underline{1320}$ ✓

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7. [3 marks]

How many numbers between 1 and 1000 are divisible by 4, 6 or 15?

$N(4) = 250$ $N(12) = 83$ $LCM(4,6) = 12$

$N(6) = 166$ $N(60) = 16$ $LCM(4,15) = 60$

$N(15) = 66$ $N(30) = 33$ $LCM(6,15) = 30$

$LCM(4,6,15) = 60$

$\text{Total} = 250 + 166 + 66 - 83 - 16 - 33 + 16$

$= 366$

8. [7 marks]

Decide whether the following statements are true or false and provide counter examples for those which are false.

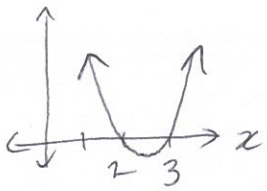
(a) For all positive numbers x, $x^2 \leq x^3$

False e.g. $x = \frac{1}{2}$ $\left(\frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)^3$

(b) $x^2 - 5x + 6 > 0$ for all $x < 0$

$(x-2)(x-3) > 0$

$x < 2$ or $x > 3$.



$\therefore$ True

(c) If $a^2 = b^2$, then $a = b$

False $a = 1, b = -1$

(d) If two quadrilaterals each have all four sides of length 5 cm, then the quadrilaterals must be congruent.

False square vs rhombus

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